LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

FIRST SEMESTER – **APRIL 2013**

MT 1818 - DIFFERENTIAL GEOMETRY

Date : 06/05/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

Answer ALL the questions
All questions carry equal marks

I a) Define tangent at any point of the space curve and derive the equation in terms of polar coordinates.

(or)

b) Prove that the curvature is the rate of change of angle of contingency with respect to arc length. [5]

c) Derive the Serret-Frenet formulae for the space curve in terms of Darboux vector.

(or)

d) Derive the equation of the osculating plane at a point on the curve of intersection of two surfaces $f(x, y, z) = 0 = g(x, y, z)$ in terms of the parameter u . [15]

II a) Define the following:

1. Curve
2. Surface
3. Pitch of the helix
4. Class over an interval
5. Parameter

(or)

b) Prove that the necessary and sufficient condition that a space curve may be helix is that the ratio of its curvature to torsion is always a constant. [5]

c) Define evolute and involute. Also find their equations.

(or)

d) State and prove the fundamental theorem of space curves. [15]

III a) Prove that the necessary and sufficient condition for the lines of curvature to be parametric curve is that $f = 0$ and $F = 0$.

(or)

b) Prove that the first fundamental form is a positive definite. [5]

c) Show that a necessary and sufficient condition for a surface to be developable is that the Gaussian curvature is zero.

(or)

d) Derive polar and tangential developables associated with a space curve. [15]

IV a) State the duality between space curve and developable.

(or)

b) Derive the geometrical interpretation of second fundamental form. [5]

c) Find the first and second fundamental form of the curve $x = a\cos\theta\sin\phi,\ y = a\sin\theta\sin\phi,\ z = a\cos\phi$. [15]

(or)

d) Prove that on a general surface, a necessary and sufficient condition for the curve $v = c$ to be a geodesic is that $EE_2 + FE_1 - 2EF_1 = 0$ for all values of the parameter.

V a) Derive Weingarton equation.

(or)

b) Show that sphere is the only surface in which all points are umbilics. [5]

c) Derive Gauss equation in terms of Christoffel's symbol.

(or)

d) State the fundamental theorem of Surface Theory and demonstrate it in the case of a unit sphere. [15]

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